Abstract

This dissertation focuses on the analysis, algorithms and applications for Structured Total Least Squares problems and includes the following original contributions.

- The different existing approaches for Structured Total Least Squares problems, the Constrained Total Least Squares, Structured Total Least Norm and Riemannian SVD approach are proven to be equivalent despite previous claims of inequality in literature. The Structured Total Least Squares problems are situated in a larger framework of so-called Structured Total Approximation Problems. This extended framework envisages other cost functions than the sum of squares as well as nonlinearly structured matrices instead of only considering affinely structured matrices. The Riemannian SVD framework is extended for nonlinearly structured matrices. The theoretical derivation of this nonlinear Structured Total Least Squares approach is illustrated with a numerical example.

- The Structured Total Least Squares problem is shown to be a nonconvex problem, suffering from many local minima. Using existing theoretical results, a new definition for nongeneric Structured Total Least Squares problems is introduced and it is shown that under certain conditions due to this nongenericity, the Structured Total Least Squares problem fails to have a solution. Theory and definitions are illustrated with new examples.
  Furthermore it is shown by means of examples that besides the local minima problem, the Structured Total Least Squares problem might also have several *optimal* solutions.

- A new projection interpretation of the Structured Total Least Squares problem is derived. Using the latter interpretation, it is possible to view Structured Total Least Squares problems as a kind of structured subspace methods, based on newly defined (structure-dependent) signal and noise subspaces.

- The nature of Structured Total Least Squares problems involving Hankel (Toeplitz) matrices is analyzed in detail, by using a specific parametrization of the data in these Hankel (Toeplitz) matrices. The difficulty of the Structured Total Least Squares problem is shown to depend on the parameters of this new parametrization.

- Existing algorithms and alternative algorithms are discussed for the Constrained Total Least Squares and the Riemannian SVD framework. For the Structured Total Least Norm framework a new algorithm is developed. The main improvement consists in the replacement of the previously used penalty function approach by the numerically robust equality constrained least squares approach. Another important improvement, given the problem of the multiple local minima, is the use of subspace based initialization methods. For Hankel (Toeplitz) matrices this is realized by an ESPRIT-type approach, but also for other structures it is possible to use a subspace based initialization method, based on the projection interpretation of the Structured Total Least Squares problem.

- Using a deconvolution example in a medical application called renography, it is demonstrated how efficient algorithms can be developed for a particular Structured Total Least Squares problem. This is done by exploiting the low displacement rank of the matrices involved in the kernel routine.

- A novel speech compression algorithm is proposed. It is a subband based scheme in which the compression of the low frequency band is essentially a Structured Total Least Squares problem. It is shown that this new scheme is related to the class of sinusoidal coders, the main difference being that damped sinusoids are used instead of undamped sinusoids. This way of encoding allows variable rate speech coding. The latter means that without having to re-encode the data, it is possible to shift to a lower bit rate –depending on the available bandwidth and/or the available computational resources– with a gradual decrease of the audible quality.

- A previously proposed framework that incorporates both the prediction error models and behavioural models is shown to be very similar to the Structured Total Least Squares problem. A novel optimization approach for this framework is derived. The latter approach provides many insight, e.g. that the constraints imposed on the model parameters have a severe impact on the obtained result. It also allows one to actually perform experiments in this new framework, using of-the-shelf optimization routines.