Kalman Filtering Techniques for System Inversion and Data Assimilation

PhD defense

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Outline

1. Introduction
   - Historical perspective
   - The Kalman filter
2. System Inversion
   - Inversion of deterministic systems
   - Inversion of combined deterministic-stochastic systems
   - Applications
3. Data Assimilation
   - Space weather
   - Suboptimal Kalman filtering
   - Space weather nowcasting example
4. Conclusions & Directions for further research

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Historical perspective

1630



Galileo Galilei

First attempt to the development of an estimation theory

1800



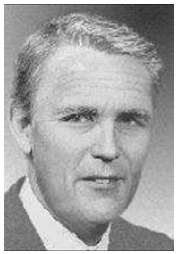
Gauss



Legendre

Least-squares estimation

1960



Kalman

Kalman filter

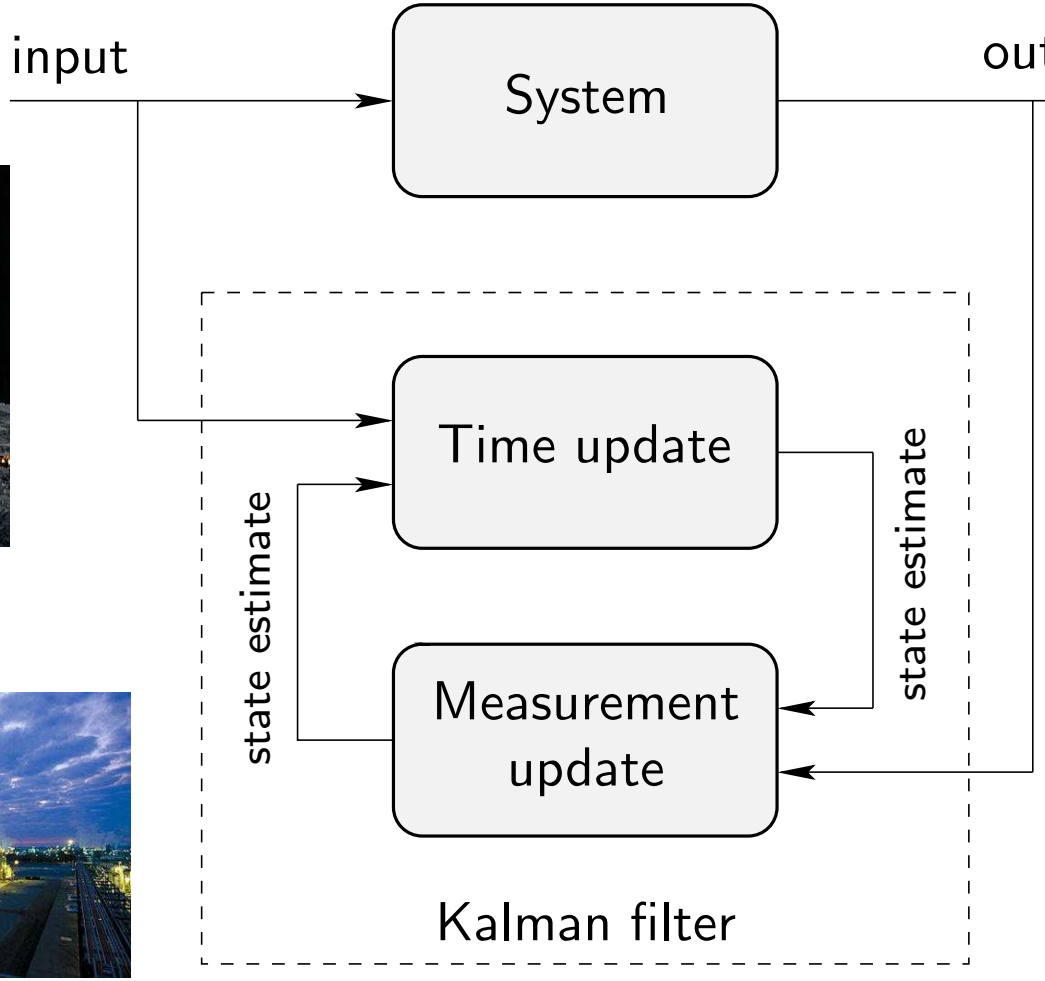
Historical perspective











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Dynamical system: input, output, state



Dynamical system: input, output, state



Input

- elevator angle
- ...

Output

- pitch angle
- flight path angle
- ...

Dynamical system: input, output, state



Input

- elevator angle
- ...

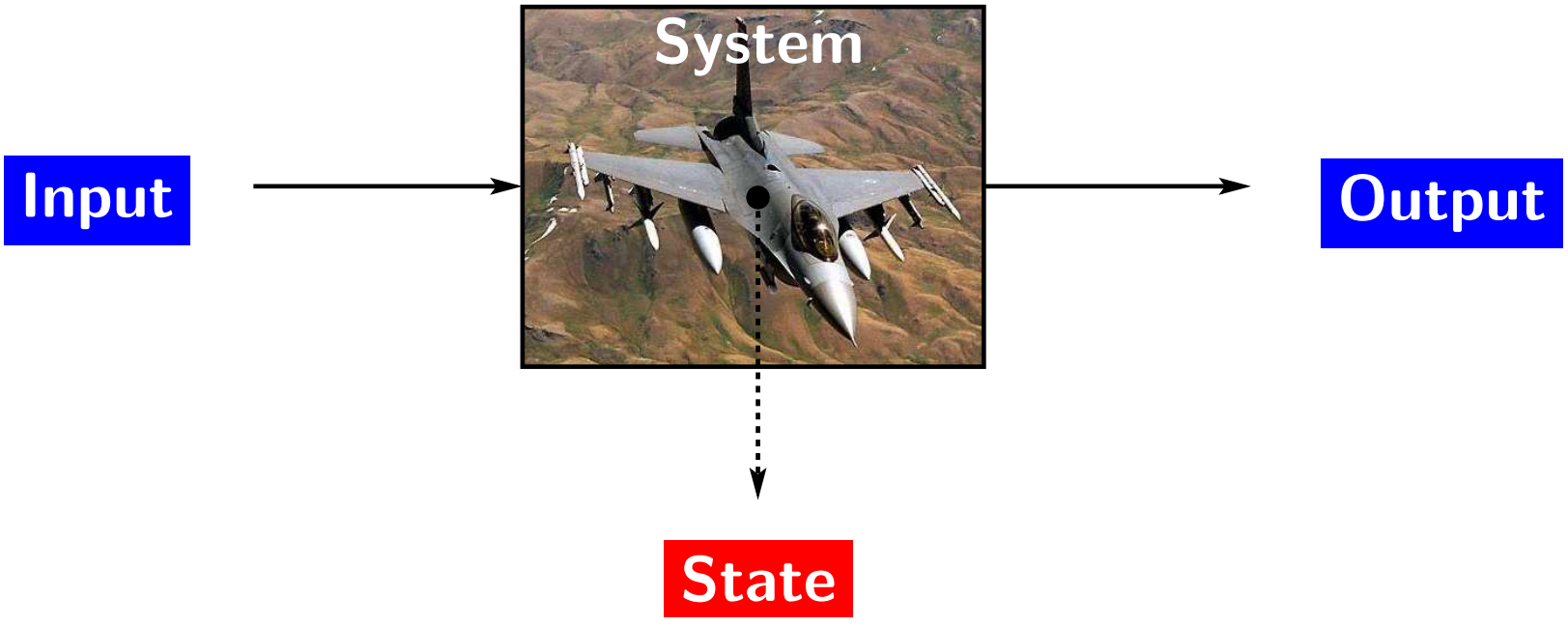
Output

- pitch angle
- flight path angle
- ...

State

- velocity
- angle of attack
- pitch angle
- pitch rate
- ...

Kalman filtering problem



Unknown
To-be-estimated

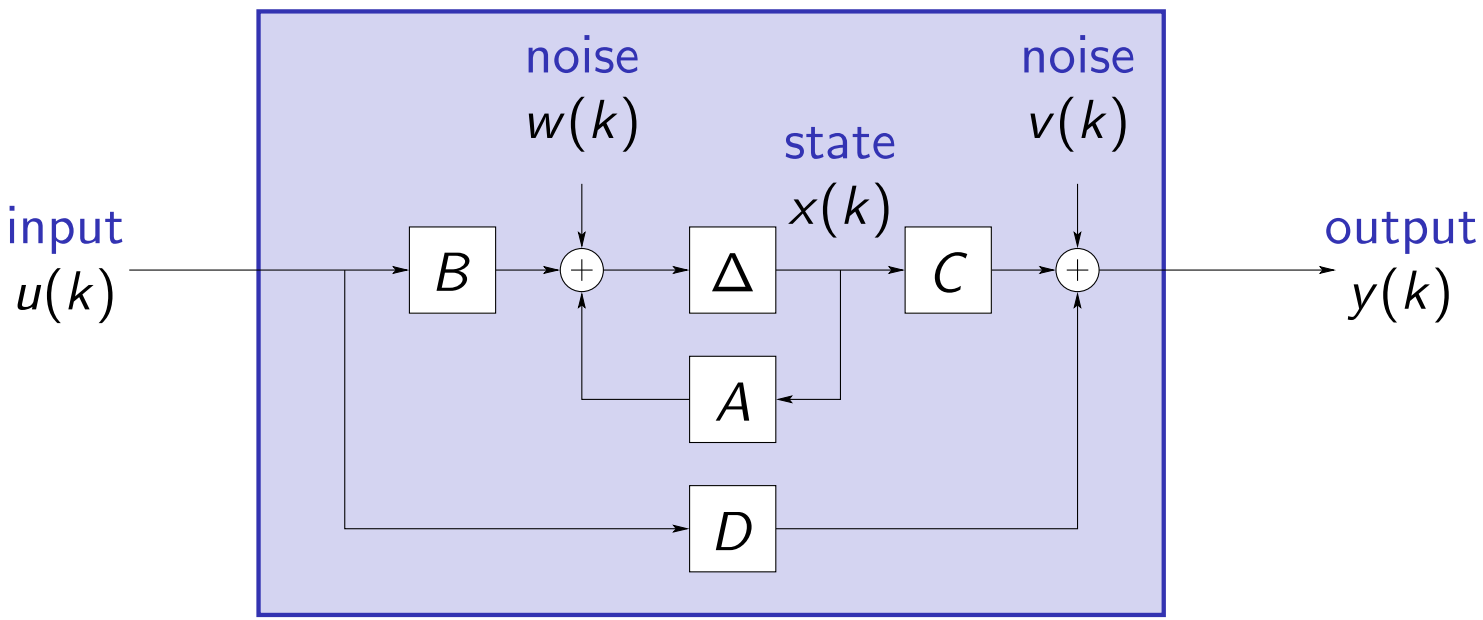
Known

Linear state-space model

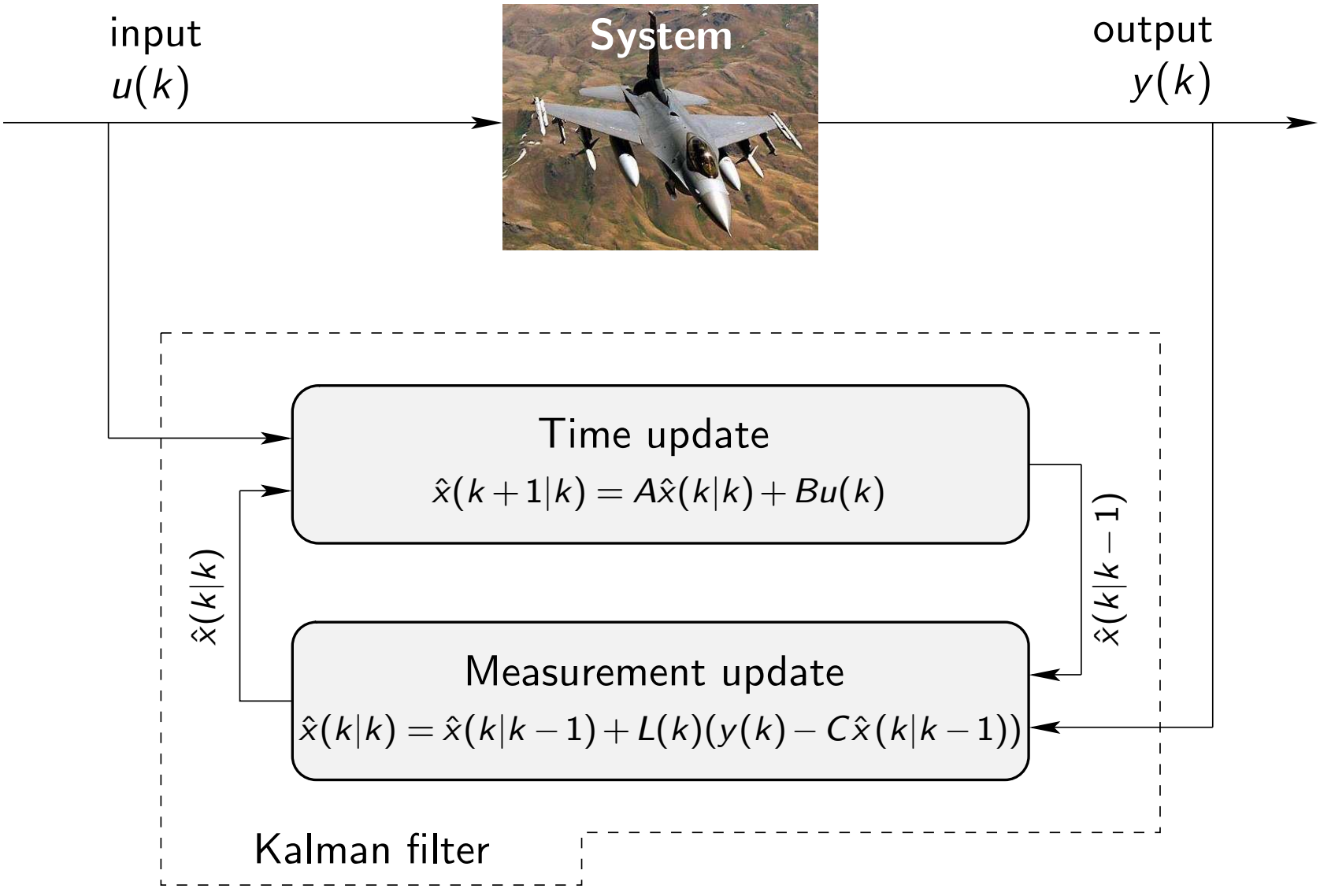
- Linear state-space model

$$x(k+1) = Ax(k) + Bu(k) + w(k)$$
$$y(k) = Cx(k) + Du(k) + v(k)$$

- Block diagram



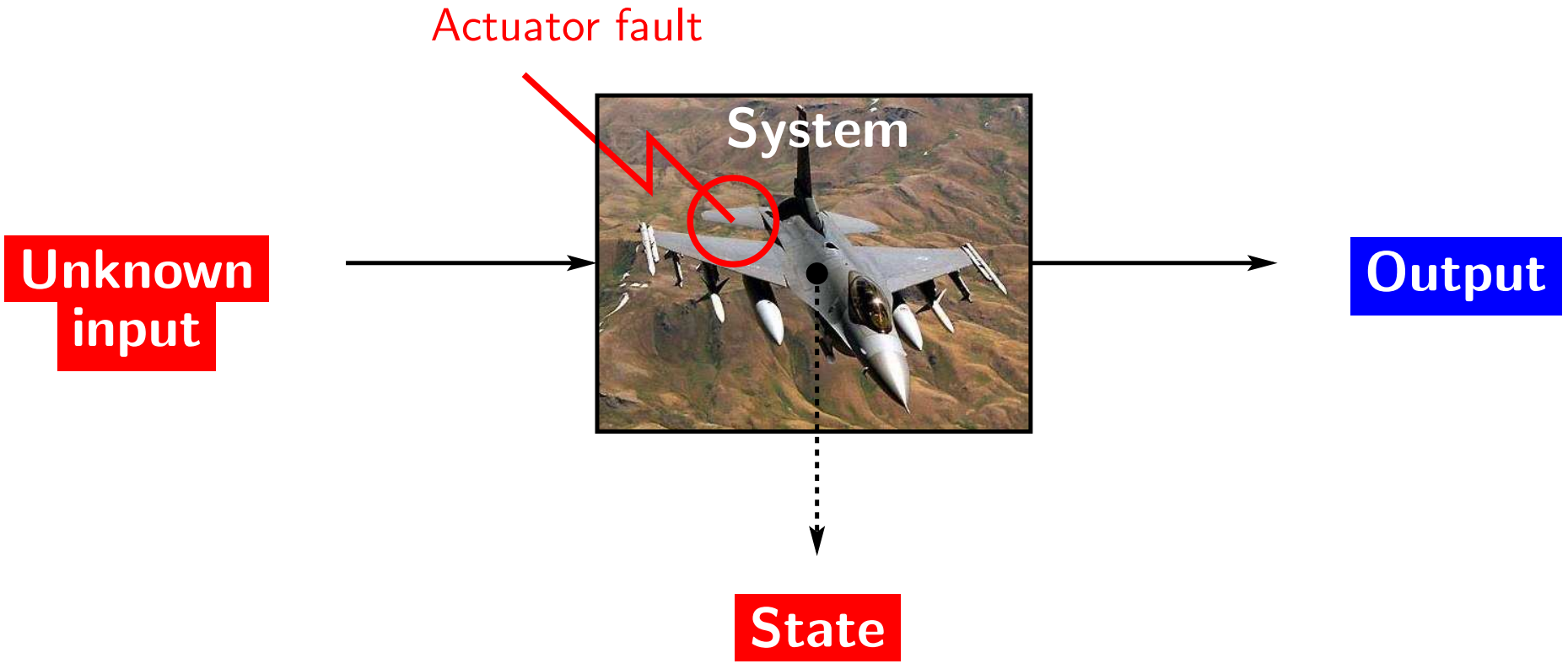
The Kalman filter



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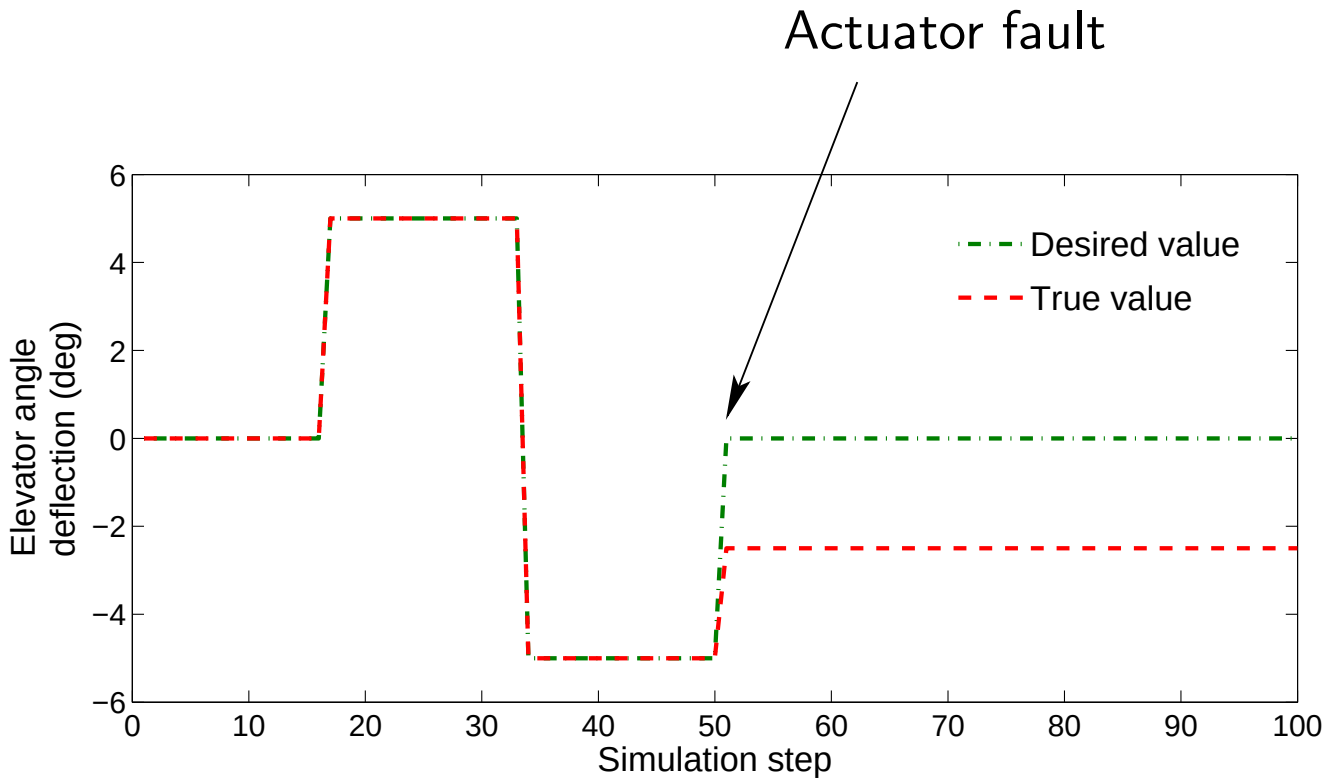
F16 example: Actuator fault



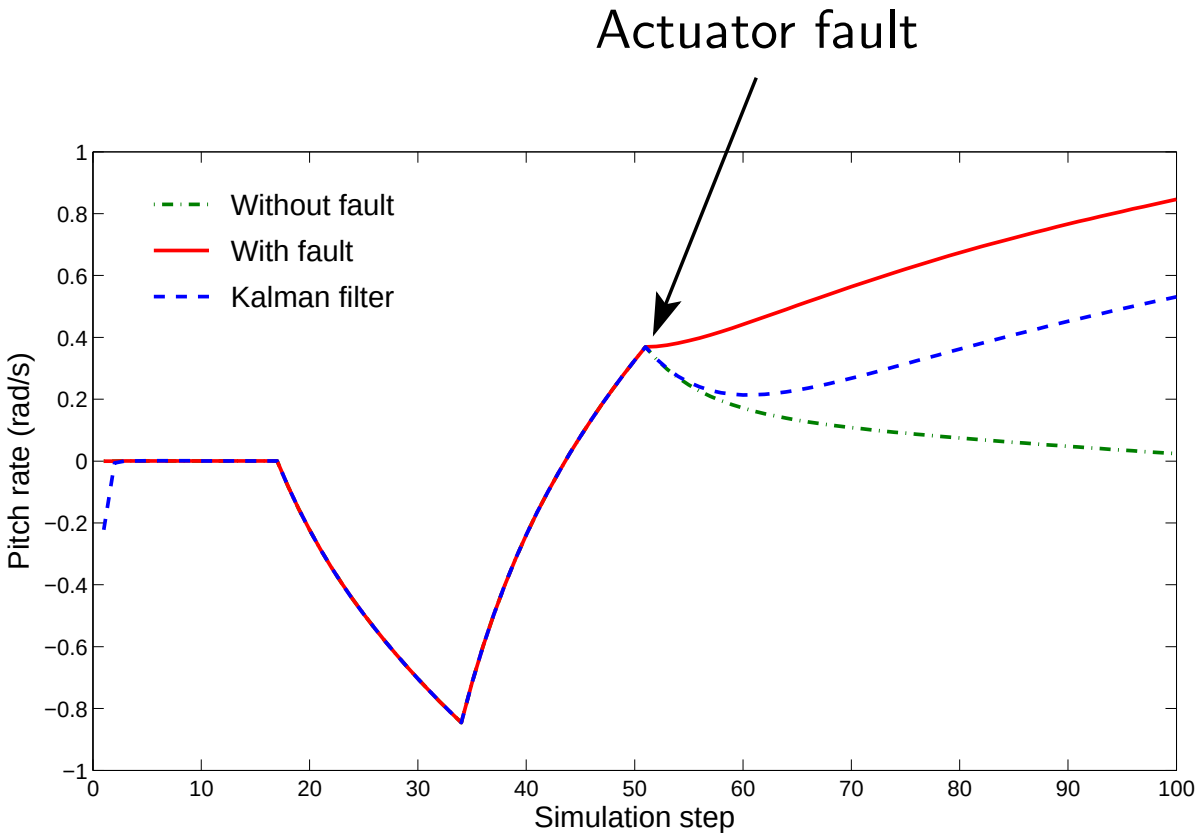
Unknown
To-be-estimated

Known

F16 example: Actuator fault

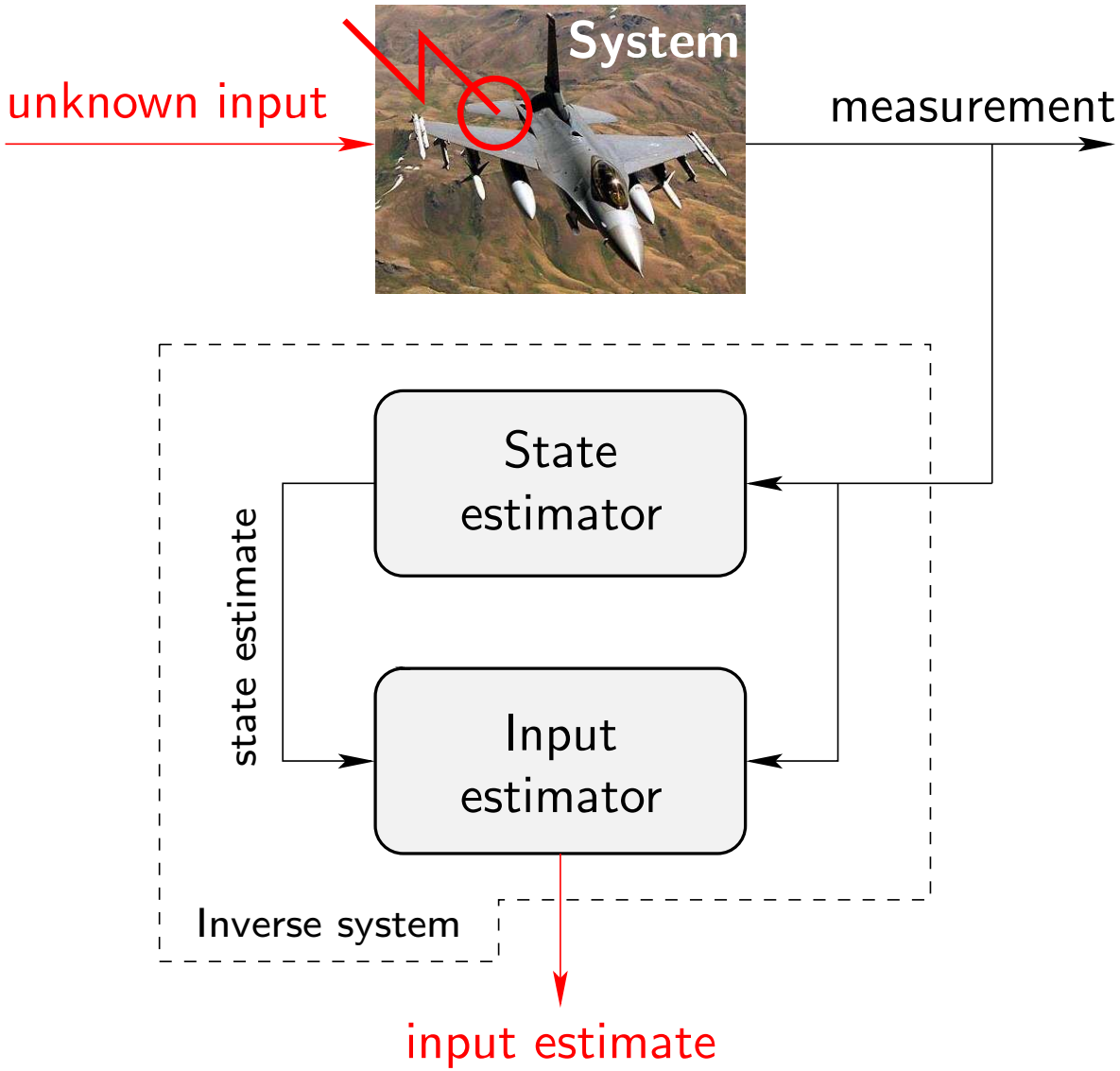


Kalman filtering in the presence of actuator faults



The Kalman filter can not deal with unknown inputs such as actuator faults

Objective


System
unknown input
measurement
State estimator
state estimate
Input estimator
Inverse system
input estimate

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History of system inversion

1965

SISO systems

Brockett & Mesarovic

first invertibility criteria and algorithms

1968-1969

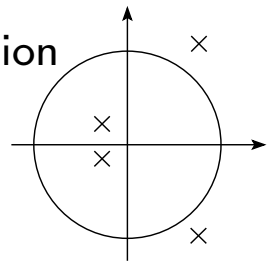
MIMO systems

Sain & Massey

time-delayed inversion

Silverman

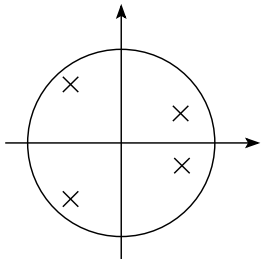
structure algorithm



1977

Stable inversion

Moylan

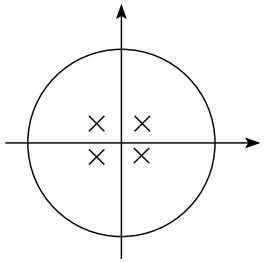


1978

Pole placement

Antsaklis

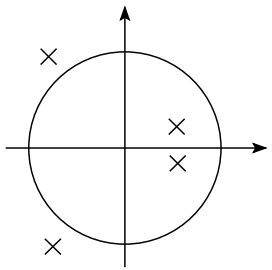
limited to instantaneous inversion

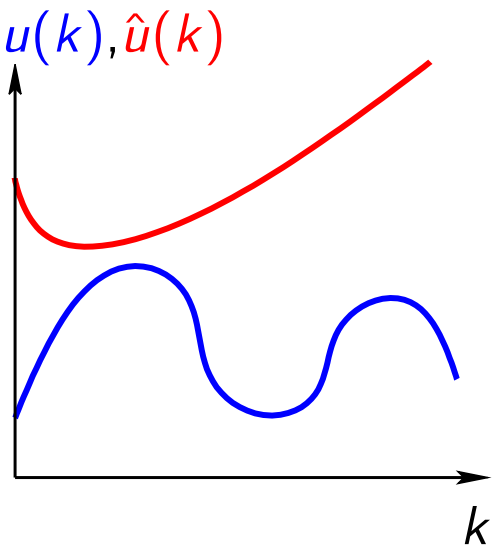


Stable inversion

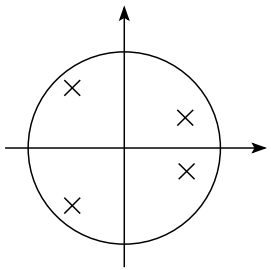
Initial state $x(0)$ unknown

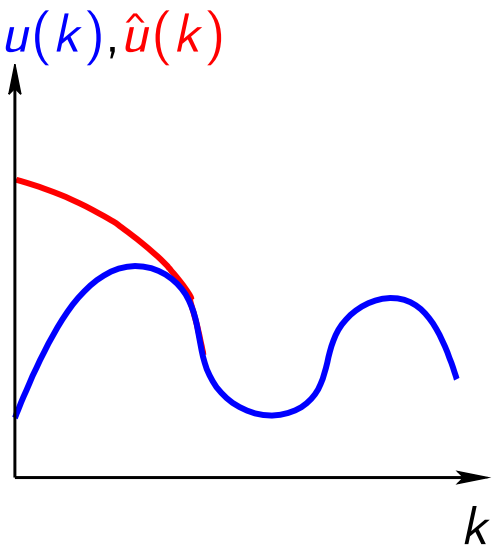
Unstable inverse



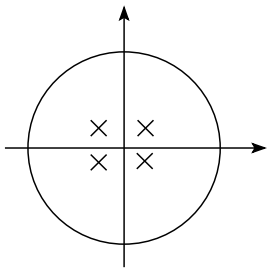


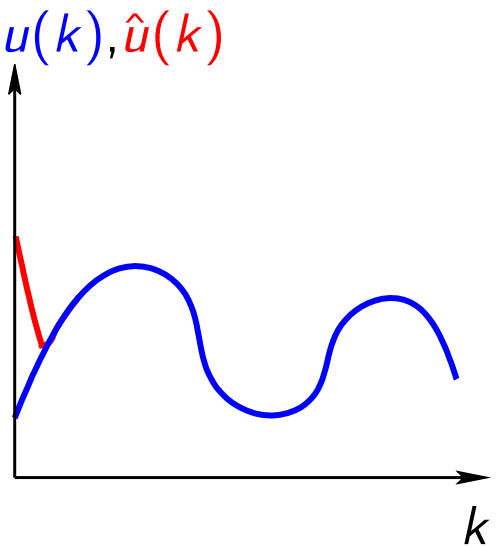
Stable inverse





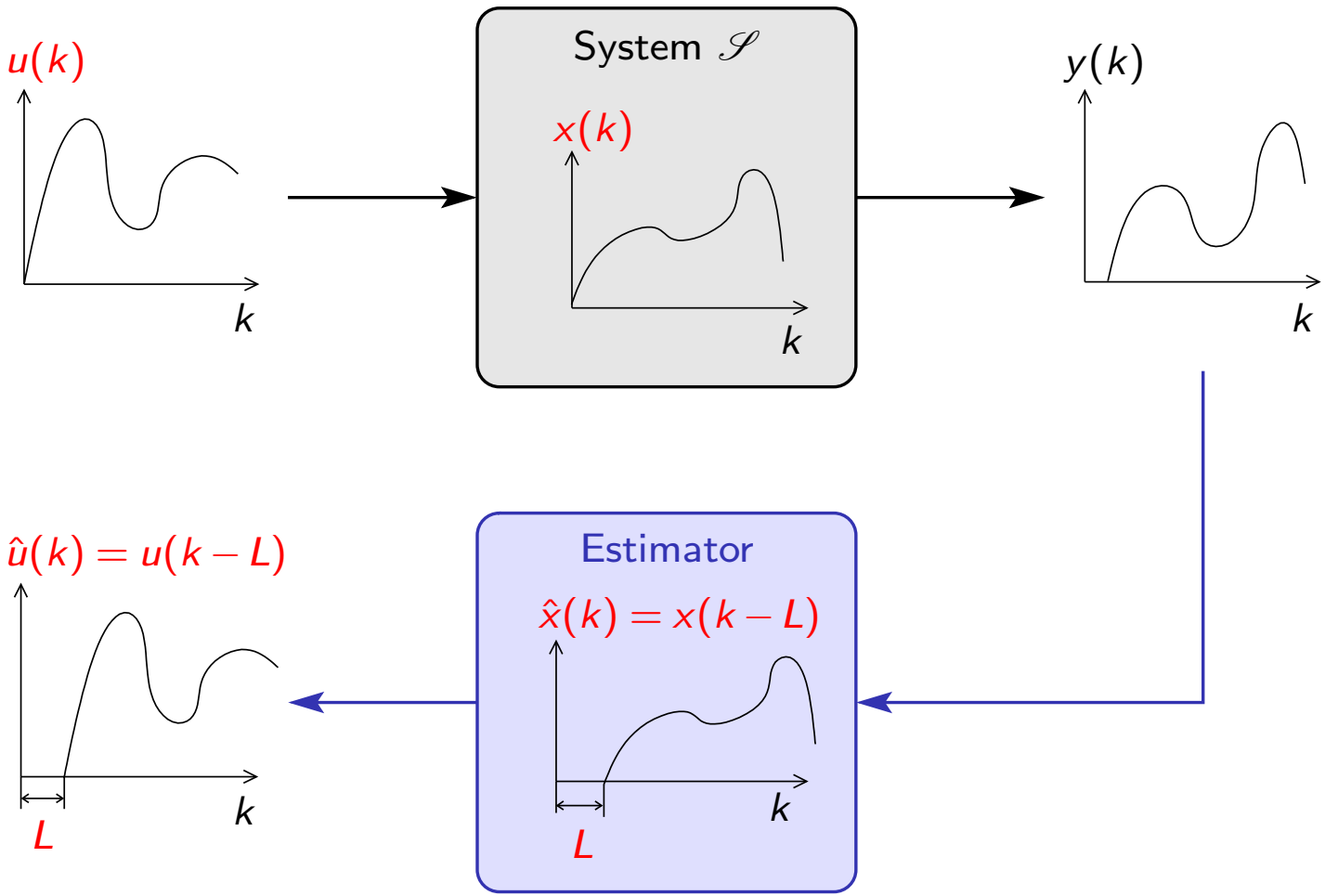
Pole placement





Time-delayed joint input-state estimation

We considered a time-delayed joint input-state estimator that can be considered as an L-delay left inverse of $\mathscr{S}$



Contributions

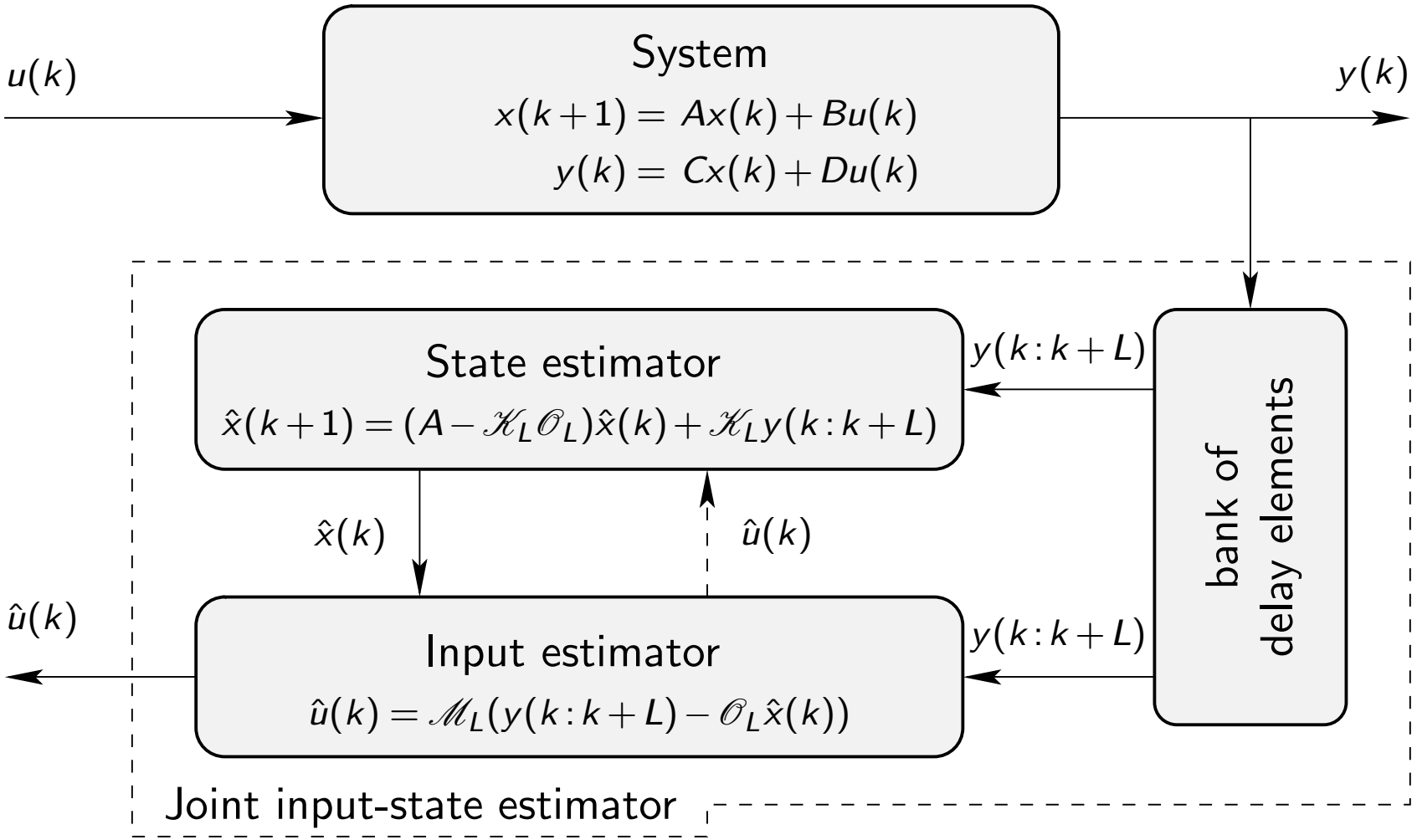
Contributions

- Derivation of a general form of a joint input-state estimator
  - General form consists of two free matrix parameters
  - If $\hat{x}(0) = x(0)$ is known, the estimator exactly reconstructs $x(k)$ and $u(k)$ for all k
- Derivation of conditions under which the free matrix parameters can be chosen so that
  - the estimator is stable

    A stable estimator exists if
    $\mathscr{S}$ has no unstable transmission zeros
  - the poles of the estimator can be placed

    The poles of the estimator can be placed if
    $\mathscr{S}$ has no transmission zeros
- Development of a new procedure to simultaneously reduce the order of the estimator and place its poles

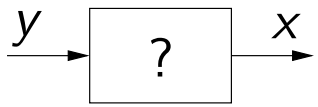
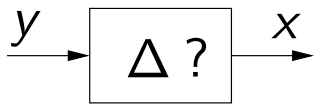
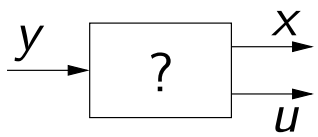
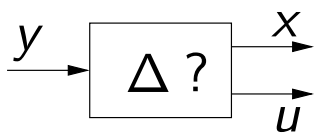
Schematic of the estimator



Outline

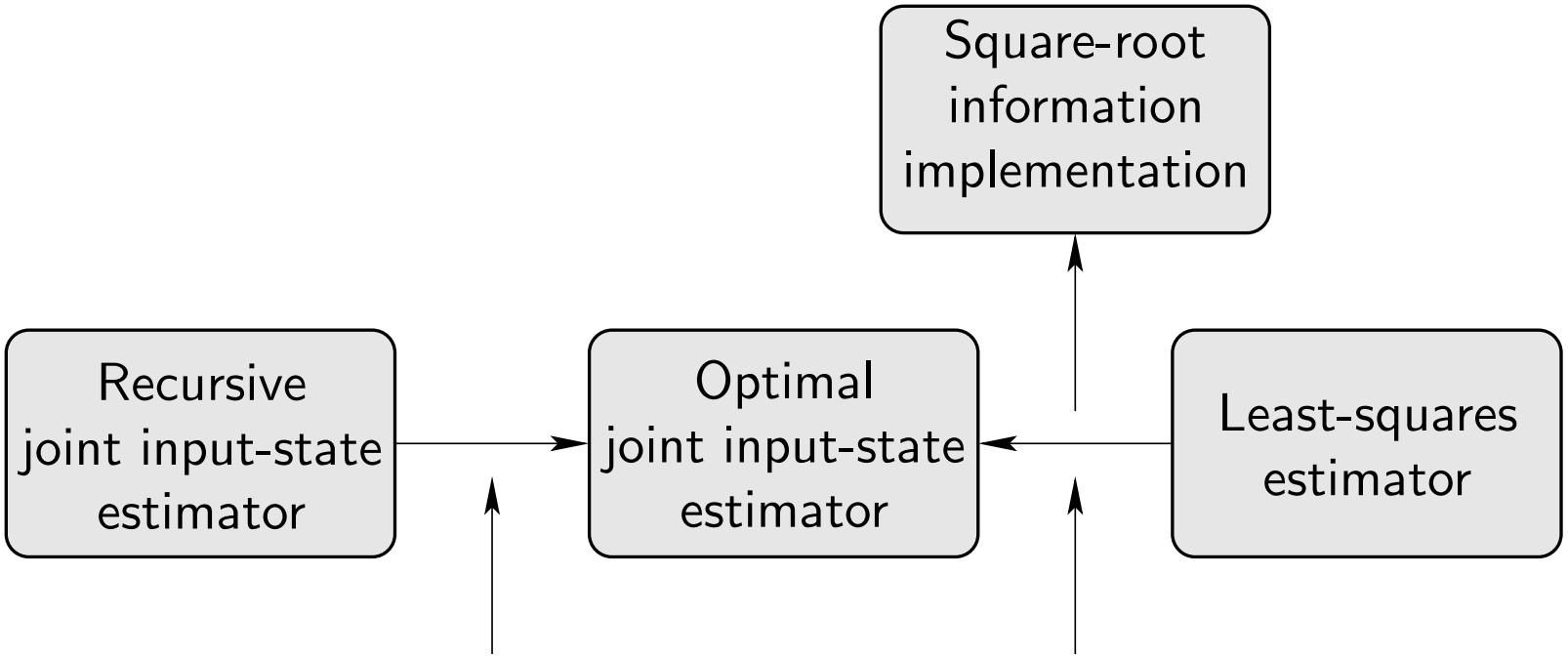
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State of the art

	Filtering $L = 0$ or $L = 1$	Time-delayed estimation
State estimation	*Kitanidis, 1987* 	*Sundaram et al., 2005* 
Joint input-state estimation	*(Hsieh, 2000)* 	

In contrast to the state estimation problem, the joint input-state estimation problem has received only little attention

Contributions



- MVU estimation
- Stochastic assumption about $w(k)$ and $v(k)$
- Cfr. interpretation of KF as optimal recursive estimator

- RLS estimation
- No stochastic assumption about $w(k)$ and $v(k)$ needed
- Cfr. interpretation of KF as RLS estimation

[Gillijns and De Moor, Automatica, Jan. 2007]
[Gillijns and De Moor, Automatica, May 2007]
[Gillijns and De Moor, IEEE TAC, To be published]

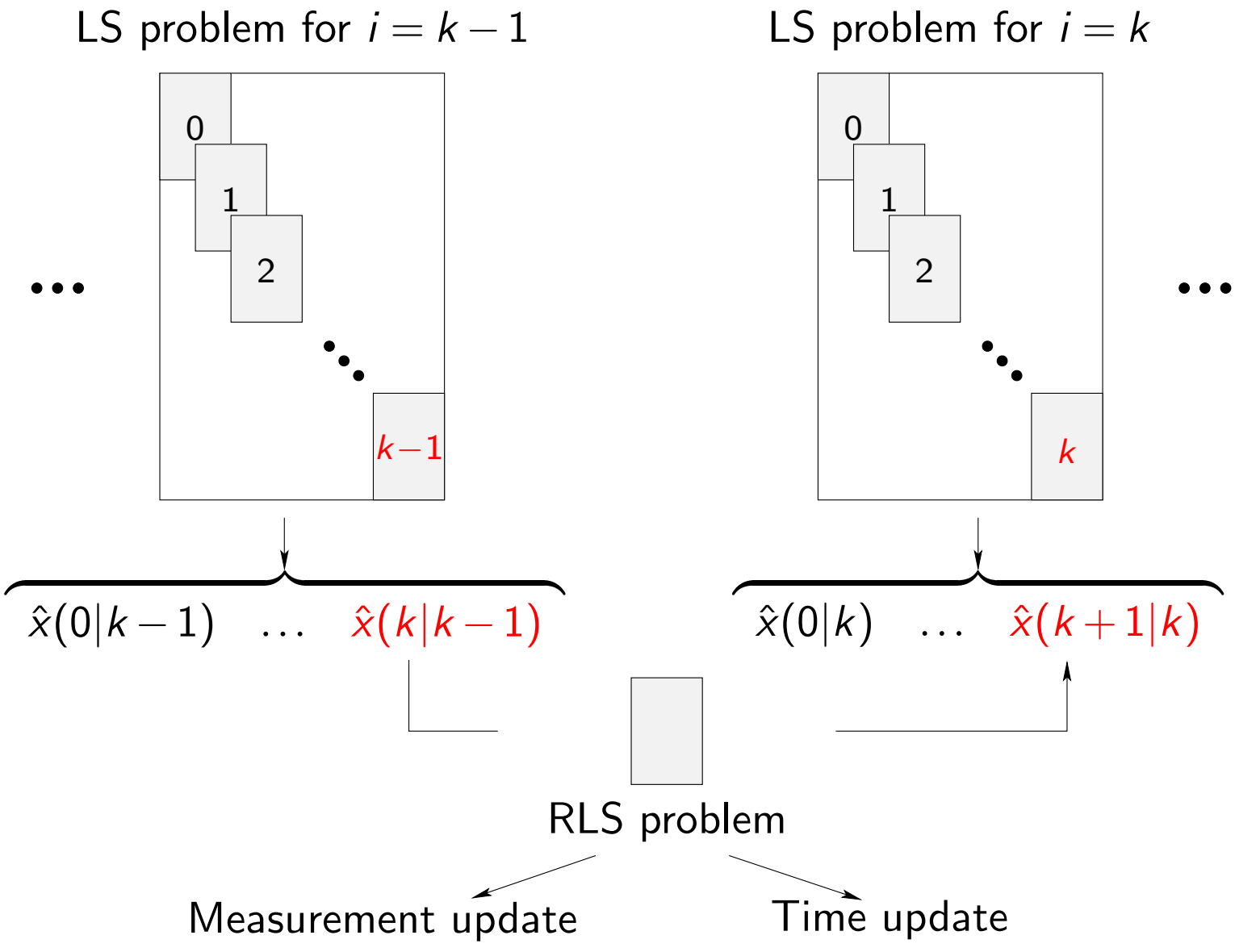
Least-squares estimation

Consider for $i = 0, 1, \ldots$ the LS problem

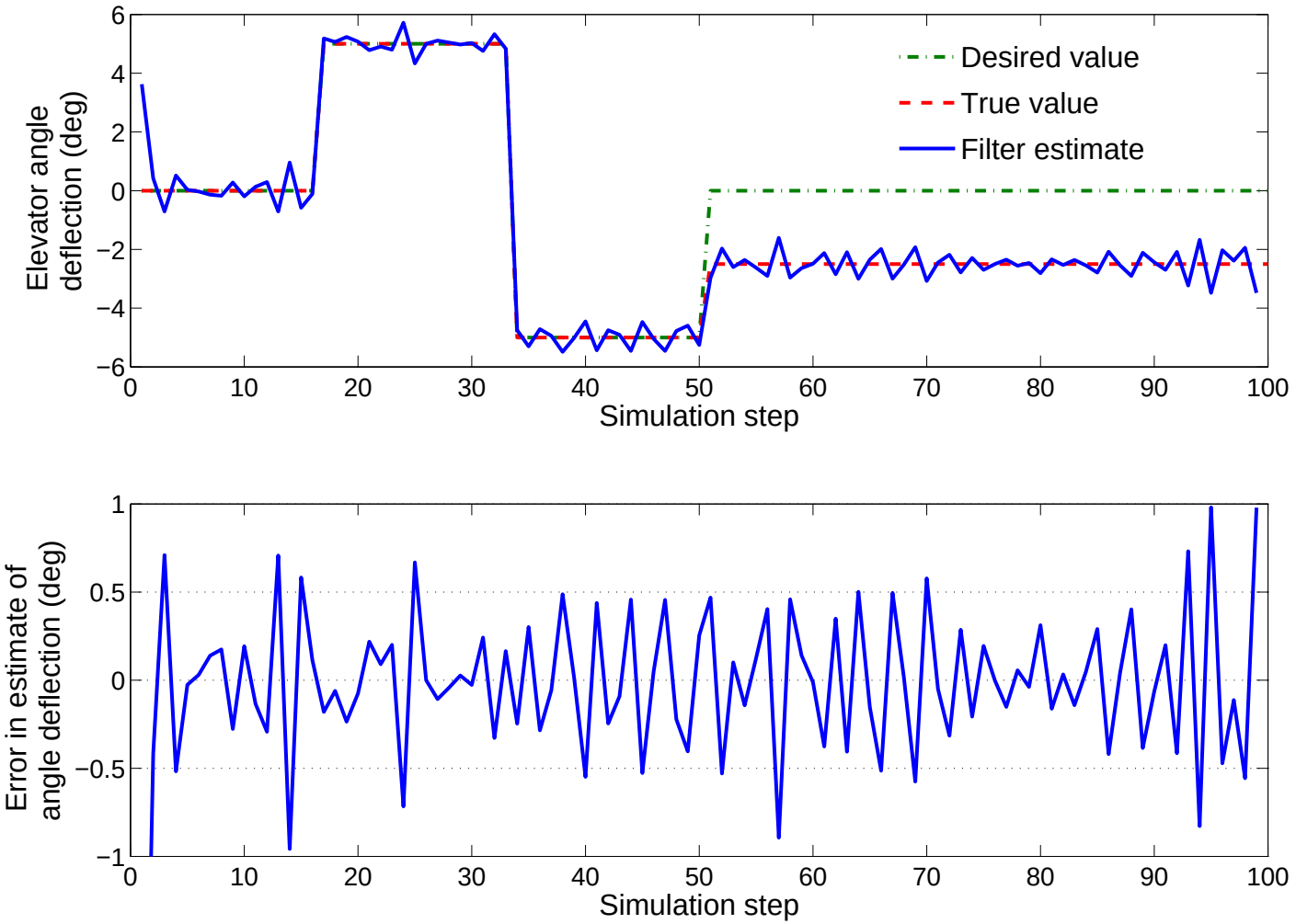
$$\min_{\substack{x(0),\ldots,x(i+1)\\ u(0),\ldots,u(i)}} \left\| \underbrace{\begin{bmatrix} \hat{x}(0|-1) \\ y(0) \\ 0 \\ \vdots \\ y(i) \\ 0 \end{bmatrix}}_{\textit{known}} - \begin{bmatrix} I & & & & & \\ C & D & & & & \\ A & B & -I & & & \\ & & \ddots & & & \\ & & & C & D & \\ & & & A & B & -I \end{bmatrix} \underbrace{\begin{bmatrix} x(0) \\ u(0) \\ \vdots \\ x(i) \\ u(i) \\ x(i+1) \end{bmatrix}}_{\textit{unknown}} \right\|^2_{W(i)}$$

where $W(i) = \mathrm{diag}(P(0|-1)^{-1}, R^{-1}, Q^{-1}, \ldots, Q^{-1})$

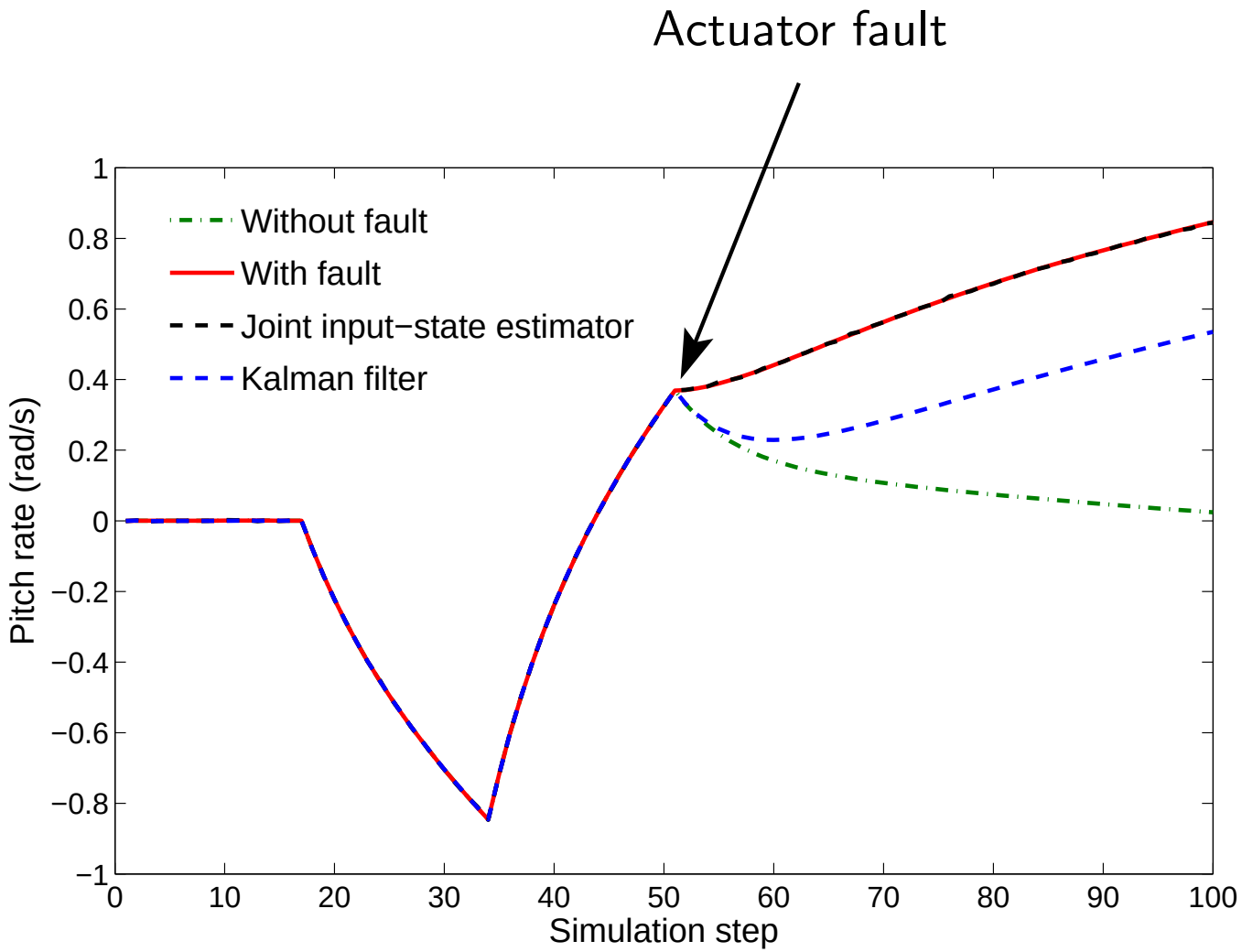
Recursive least-squares filtering



F16 example


Desired value
True value
Filter estimate
Elevator angle deflection (deg)
Simulation step
Error in estimate of angle deflection (deg)
Simulation step

F16 example

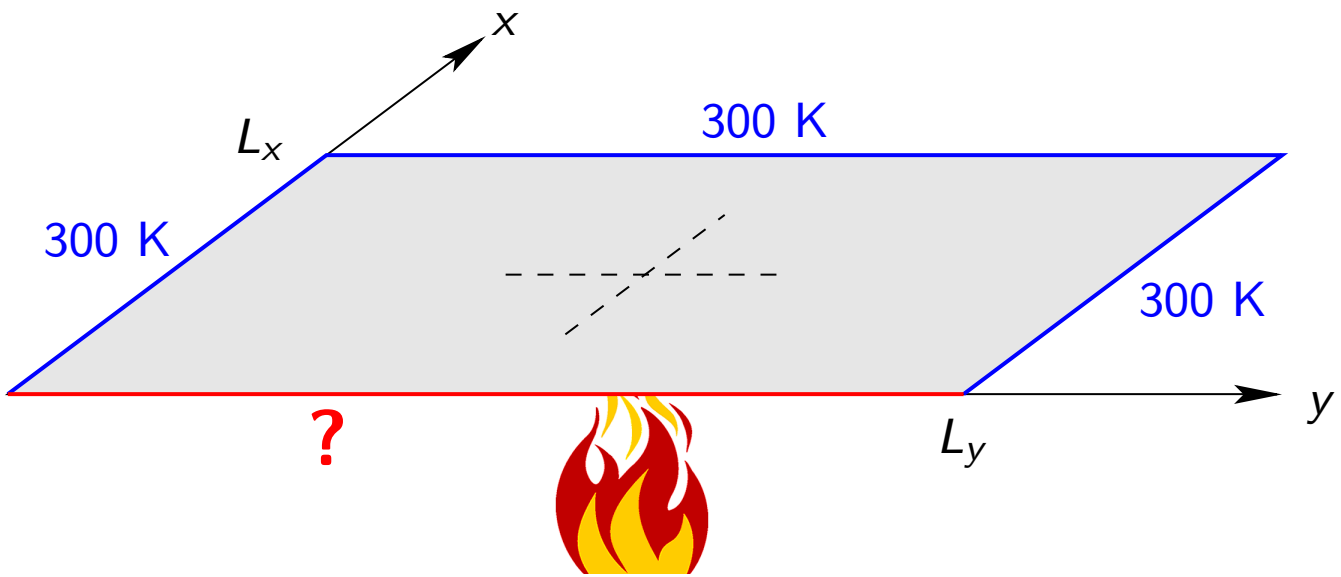

Actuator fault
Without fault
With fault
Joint input−state estimator
Kalman filter
Pitch rate (rad/s)
Simulation step
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
10
20
30
40
50
60
70
80
90
100

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Boundary condition estimation

- Setup
  - Consider heat conduction in a plate with dimensions $L_x \times L_y$



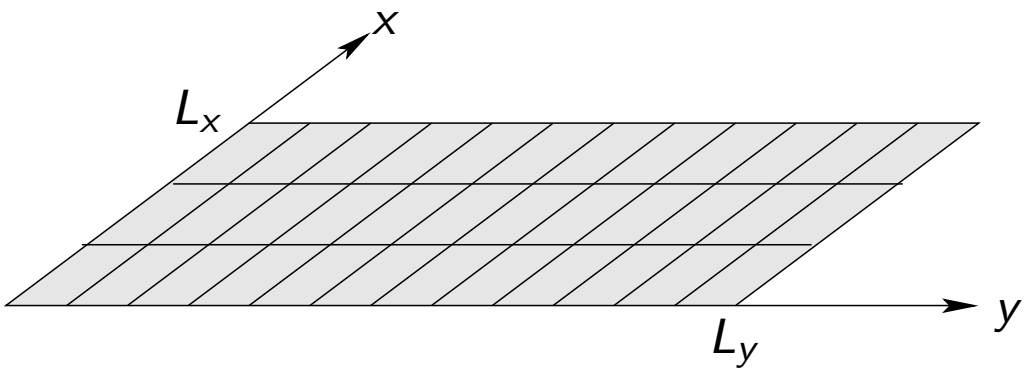
  - Problem formulation
    Simultaneously estimate the temperature $T(x, y, t)$ and the unknown boundary condition at $x = 0$

Boundary condition estimation

- Problem solution
  - Heat conduction is governed by the PDE

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + u_f$$

    with T the temperature and u_f the heat input due to the flame
  - Discretization yields state space model

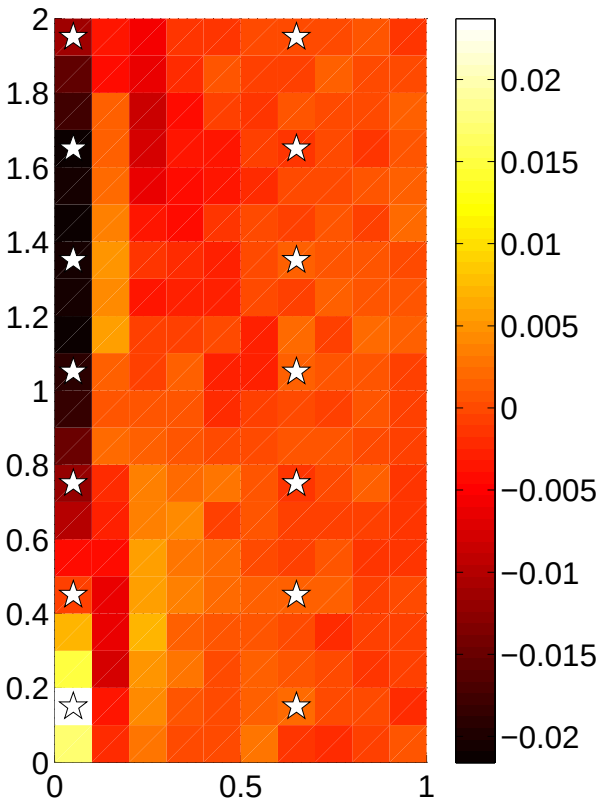


  - Expand the unknown boundary condition $u(k)$ as

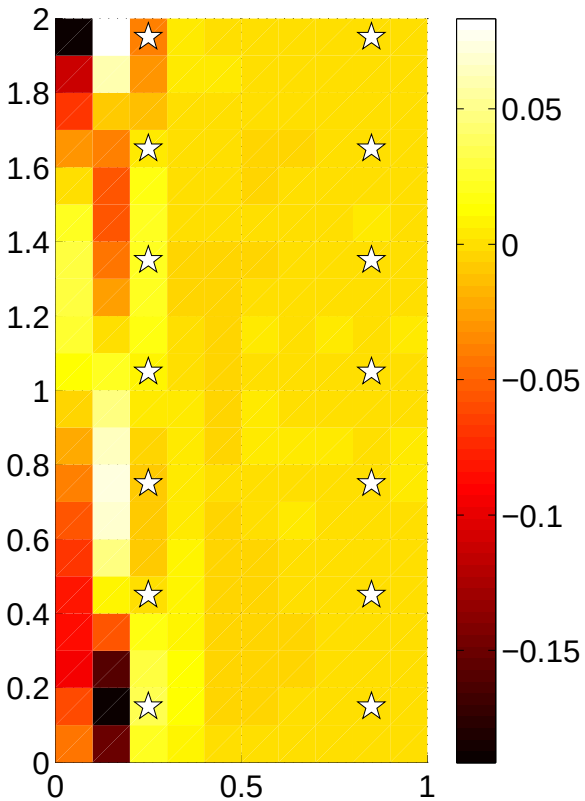
$$u(k) = \sum_{i=1}^{N} a_i(k)\phi_i$$

  - Simultaneously estimate the state vector and the vector of unknown coefficients $a(k) := [a_1(k)\ a_2(k)\ \dots\ a_N(k)]^T$

Simulation results



Filtering



Time delayed estimation

[Gillijns and De Moor, 2007]

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Space weather

- Solar wind
  - A constant stream of plasma emitted by the Sun
  - The speed of the solar wind is not constant
    - average speed is around 500 km/s
    - speed can reach up to 2500 km/s when a CME is superimposed
- Space weather
  - Effects resulting from perturbations in the speed of the solar wind
  - Can harm astronauts in space, damage satellites, disrupt longe range communications, create power blackouts,...

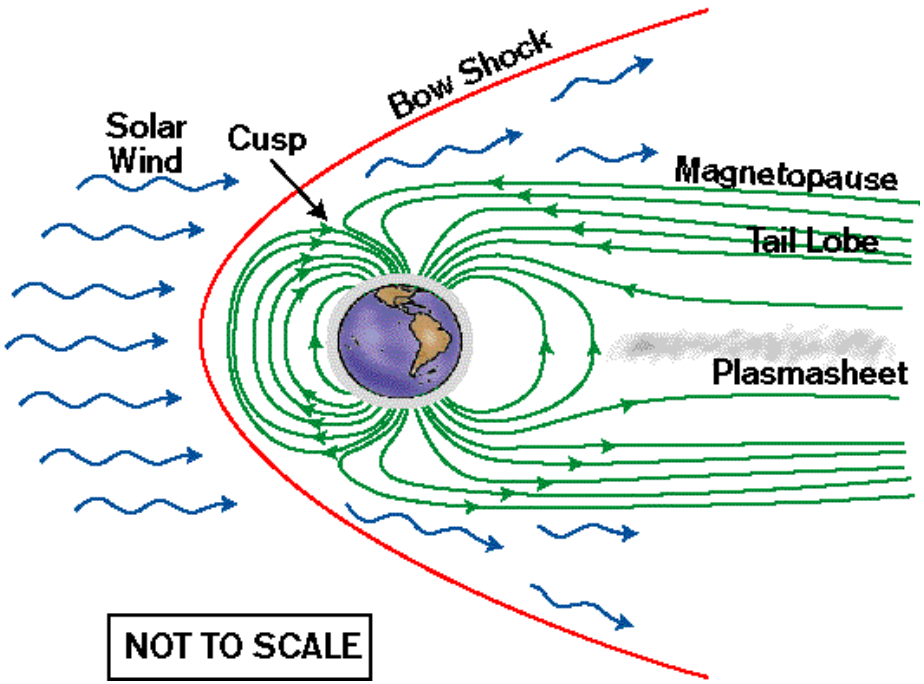
Bow shock induced by the solar wind

The solar wind travels at supersonic speed

$\Rightarrow$ a bow shock is formed when it encounters an obstacle like the Earth

(cfr. the shock wave formed by an airplane that breaks the sound barrier)

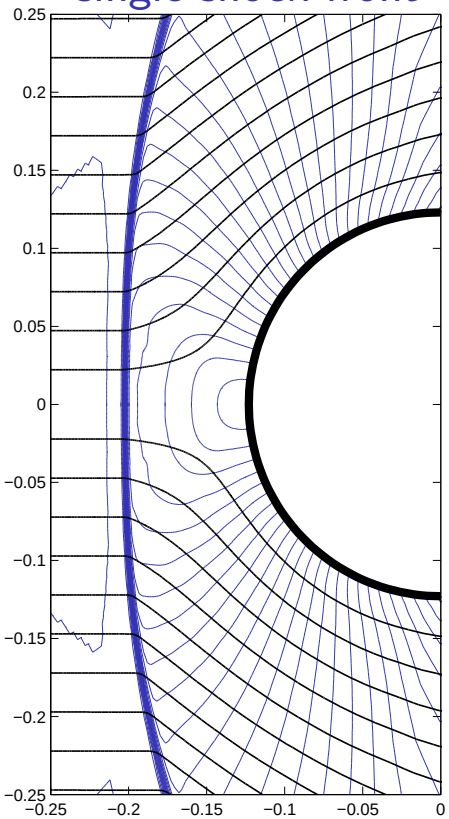




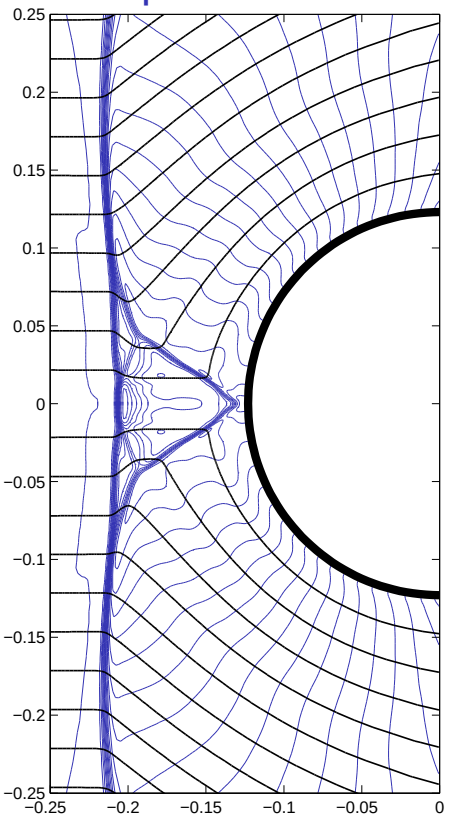
Topology of the bow shock

The topology of the bow shock changes significantly with the properties of the incoming plasma flow

Pressure dominated
single shock front



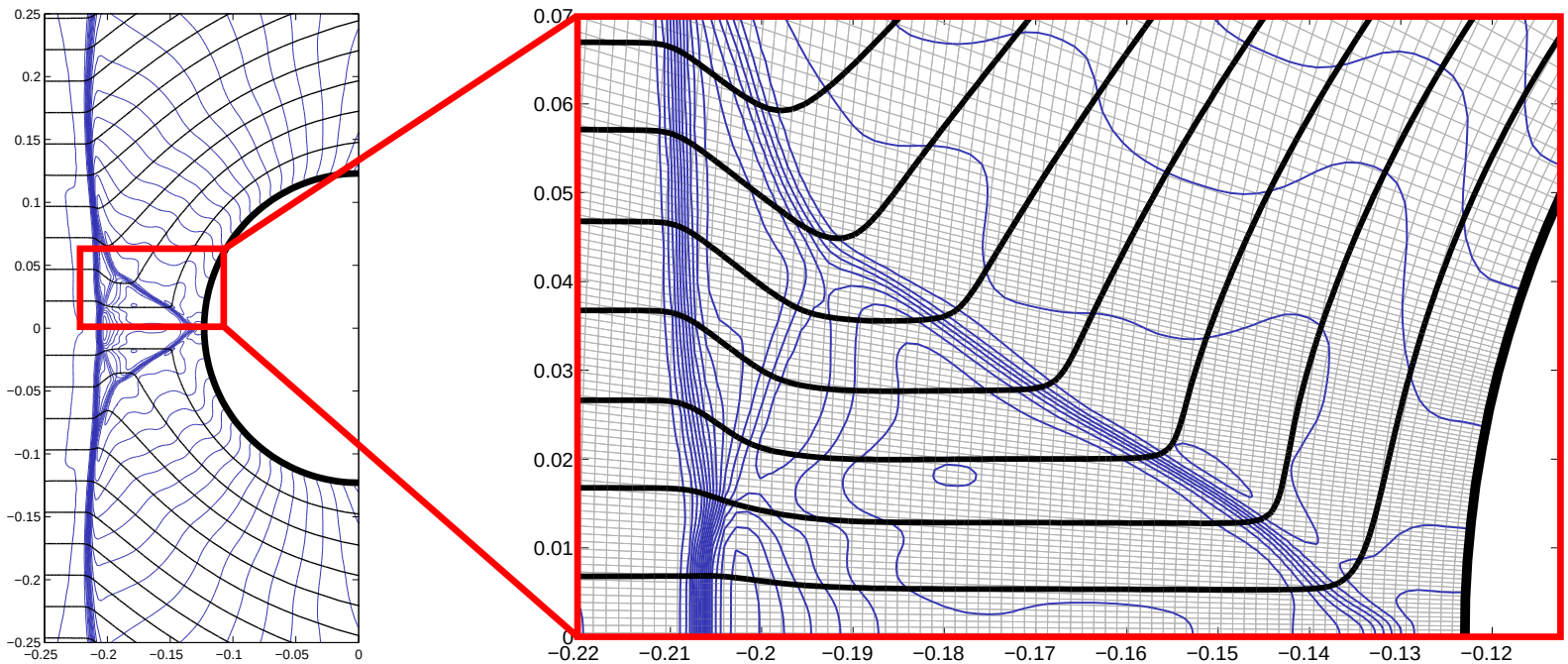
Magnetically dominated
multiple shock fronts



[De Sterck et al., 1998]

Modeling the bow shock

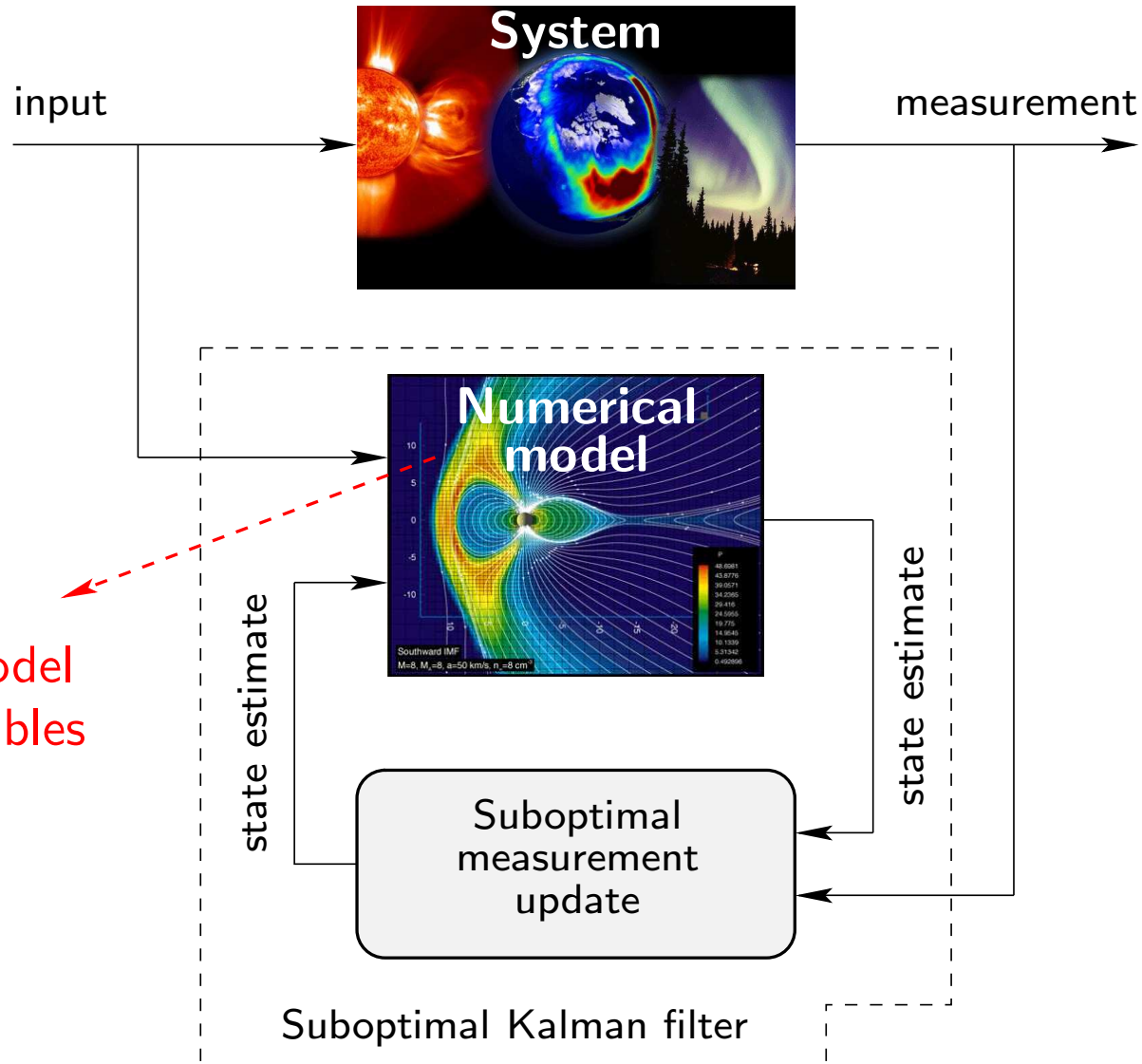
- Plasma flow is governed by the MHD equations
- For numerical simulation, the eqs. are discretized over a huge grid
  - Number of grid cells: 15376
  - Dimension of state vector: 92256



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Data assimilation



Suboptimal Kalman filtering

- The KF is not directly applicable in DA due to
  - its high computational cost (years of computation !) & immense storage requirements (terabytes !)
  - its limitation to linear models

  $\Rightarrow$ Suboptimal approximations of the KF needed

- DA for space weather application is very challenging due to
  - the sparseness of observations

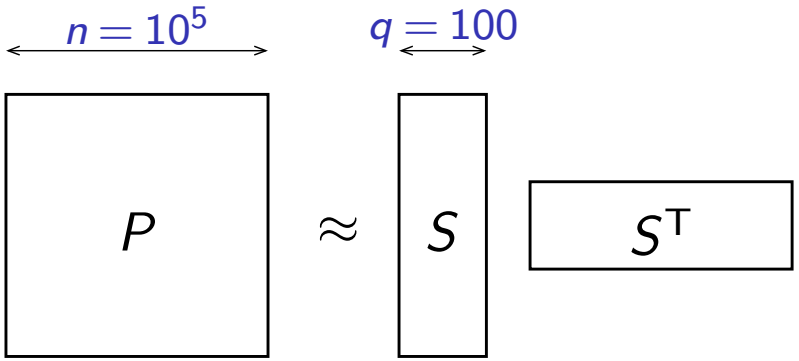
    measurements from less than 10 satellites available
    $\Leftrightarrow$ Earthly weather prediction: approximately 10^5 data points
  - the complexity of the numerical models

    combination of Navier-Stokes and Maxwell's eqs.
    $\Rightarrow$ highly nonlinear

  $\Rightarrow$ Advanced DA techniques needed

Reduced rank square-root filtering

- Idea
  - Propagate a square-root of the error covariance matrix
  - Make a lower rank approximation of the error covariance matrix using an SVD



$\Rightarrow$ Huge decrease in computational complexity

- Properties
  - Equivalent to the KF for $q = n$
  - Computed error covariance matrix positive semi-definite at all times

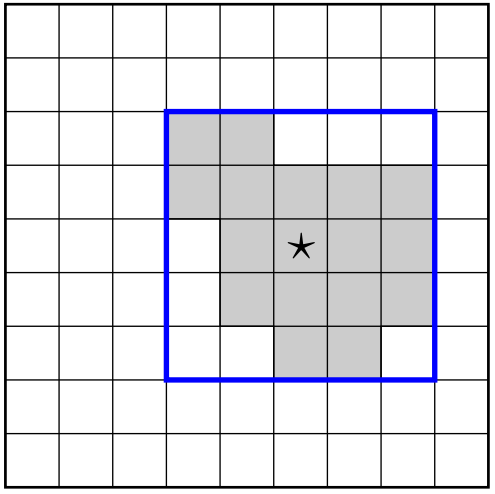
[Verlaan and Heemink, 1997]

Spatially localized filtering

- **Idea**

  Correlation between grid cells drops quickly with distance

  $\Rightarrow$ Cells greatly distant from the measurement point must not be updated



★ Measurement location

Cells of which the correlation with the measurement is above a certain threshold

Region in which the cells are updated by the measurement; Determines $\Psi(k)$

- **Implementation**

  The measurement update of the SLKF takes the form

$$\hat{x}(k|k) = \hat{x}(k|k-1) + \Psi(k)L(k)\left[y(k) - C(k)\hat{x}(k|k-1)\right]$$

  where $\Psi(k)$ constrains the update

[Barrero et al., 2005]

The RRSLSQRT filter

Contribution

The development of a suboptimal Kalman filter that combines the following ideas

- Reduced rank square-root filtering
- Spatially localized filtering
- Sequential updating

- Sequential updating
  - Translates the sparseness of observations into numerical efficiency
  - Allows to determine a different localization matrix $\Psi(k)$ for each of the measurements

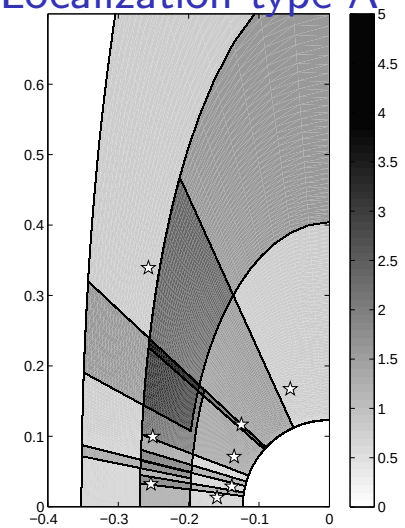
Outline

1. Introduction
   - Historical perspective
   - The Kalman filter
2. System Inversion
   - Inversion of deterministic systems
   - Inversion of combined deterministic-stochastic systems
   - Applications
3. Data Assimilation
   - Space weather
   - Suboptimal Kalman filtering
   - Space weather nowcasting example
4. Conclusions & Directions for further research

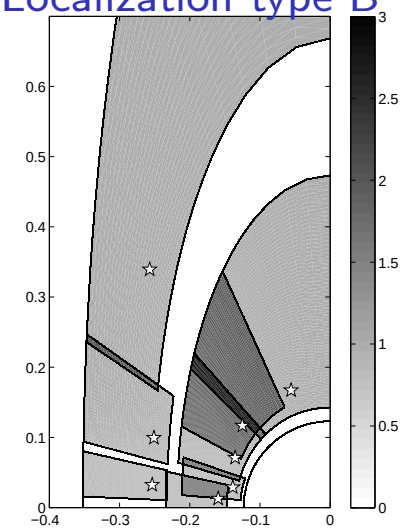
Overview of the experiments

- Objectives
  - Assess the capability of suboptimal KF techniques in space weather
  - Assess the influence of several parameters
- Parameters
  - Rank of error covariance matrix: $75, 150, 250$
  - Number of measurements: $4, 8$
  - Boundary conditions: known and unknown
  - 2 types of spatial localization

Localization type A



Localization type B



Simulation results

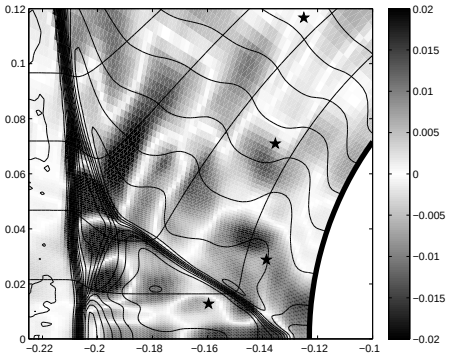
Estimation error

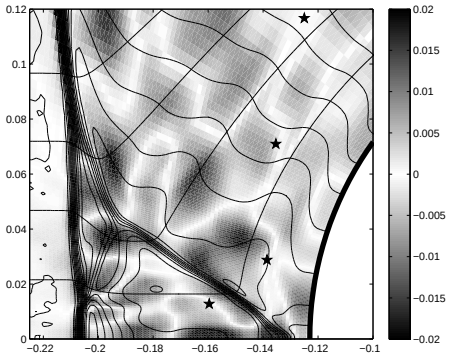
No Localization

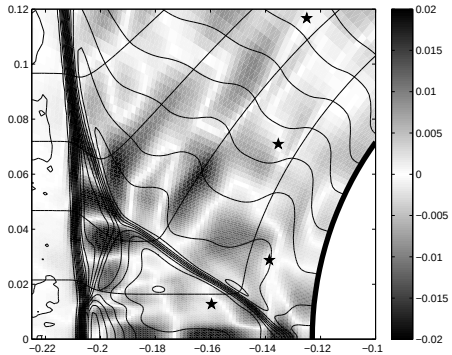
Localization type A

Localization type B

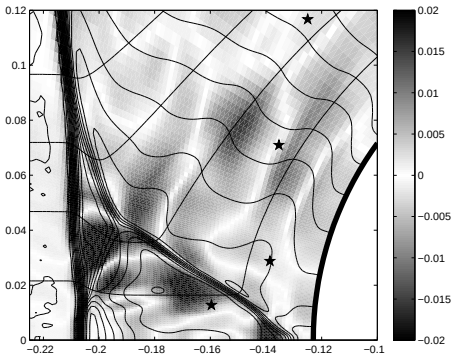
$k = 300$

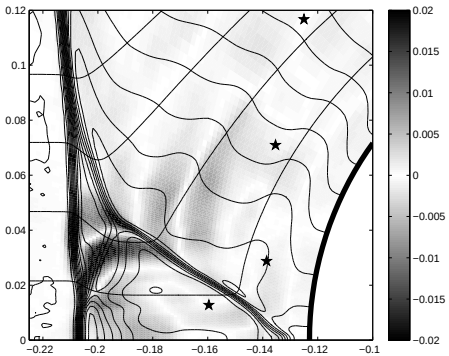


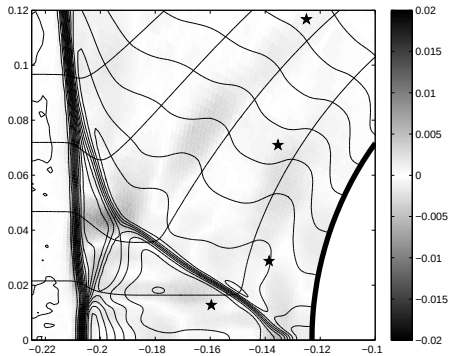




$k = 1000$

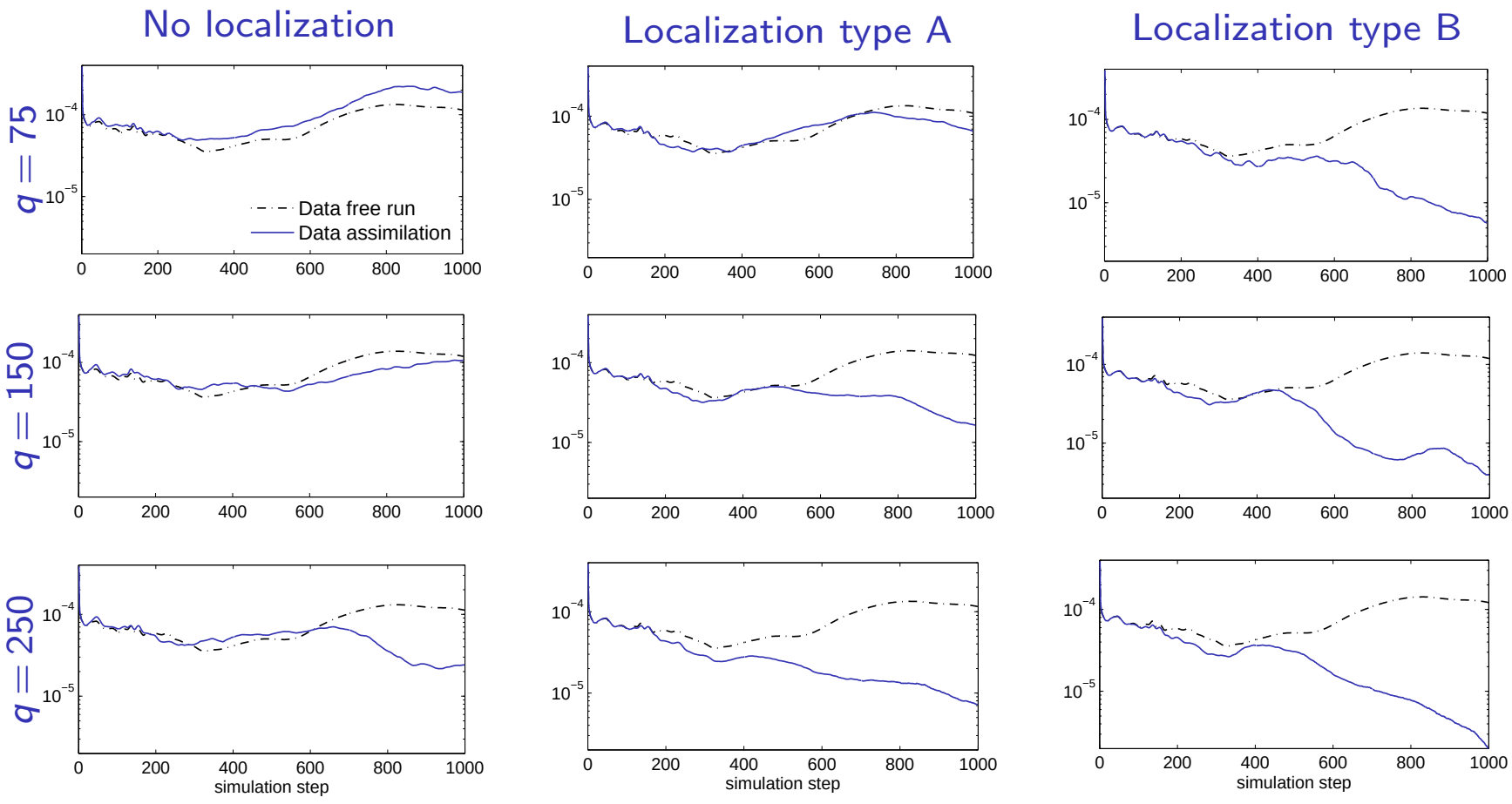






Simulation results

Mean-squared error



Outline

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Conclusions

- System inversion
  - Based on joint input-state estimation, a new procedure for left inversion of LTI deterministic system was introduced
  - The inversion procedure was extended to combined deterministic-stochastic systems. An important contribution is the establishment of a relation to LS estimation
  - The inversion procedure was employed in four applications
- Data assimilation
  - A spatially localized reduced rank filter was developed
  - The effectiveness of the reduced rank filter has been shown in a large-scale space weather nowcasting example that emulates the bow shock formed by the solar wind
  - Simulation results show that the method outperforms a data-free simulation

Directions for further research

- System inversion
  - Establish stability and convergence conditions
  - Is there a notion of ill-conditioning in system inversion?
  - Investigate the relation to LS estimation in more detail
- Data assimilation
  - Consider simulations with more complex models
  - Estimate solar wind conditions at the Sun i.s.o at the Earth
  - Consider simulations with real measurements

List of journal publications

- S. Gillijns and B. De Moor. Unbiased minimum-variance input and state estimation for linear discrete-time systems. *Automatica*, 43(1), 111–116, 2007.
- S. Gillijns and B. De Moor. Model error estimation in ensemble data assimilation. *Nonlinear Processes in Geophysics*, 14(1), 59–71, 2007.
- S. Gillijns and B. De Moor. Unbiased minimum-variance input and state estimation for linear discrete-time systems with direct feedthrough. *Automatica*, 43(5), 934–937, 2007.
- S. Gillijns and B. De Moor. Recursive least-squares estimation for systems with unknown inputs. Accepted for publication in *IEEE Transactions on Automatic Control*.